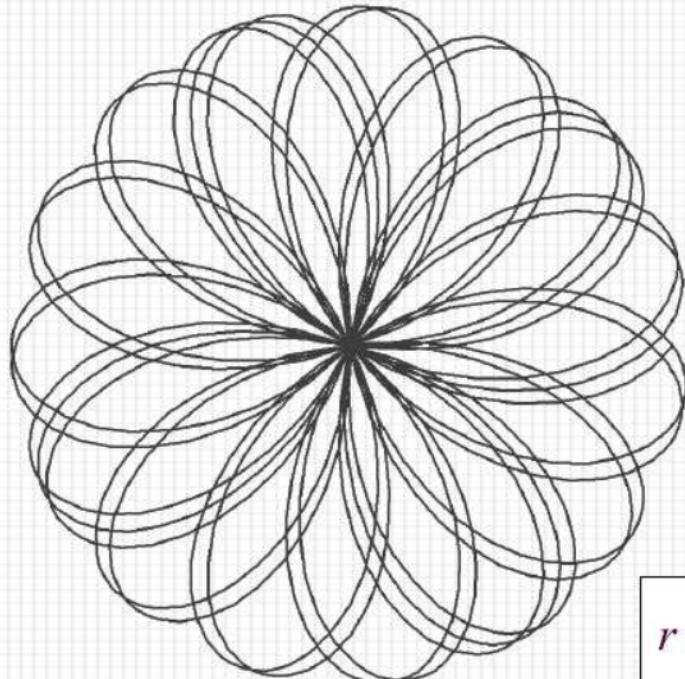


10.6: The Calculus of Polar Curves – Area Part 2

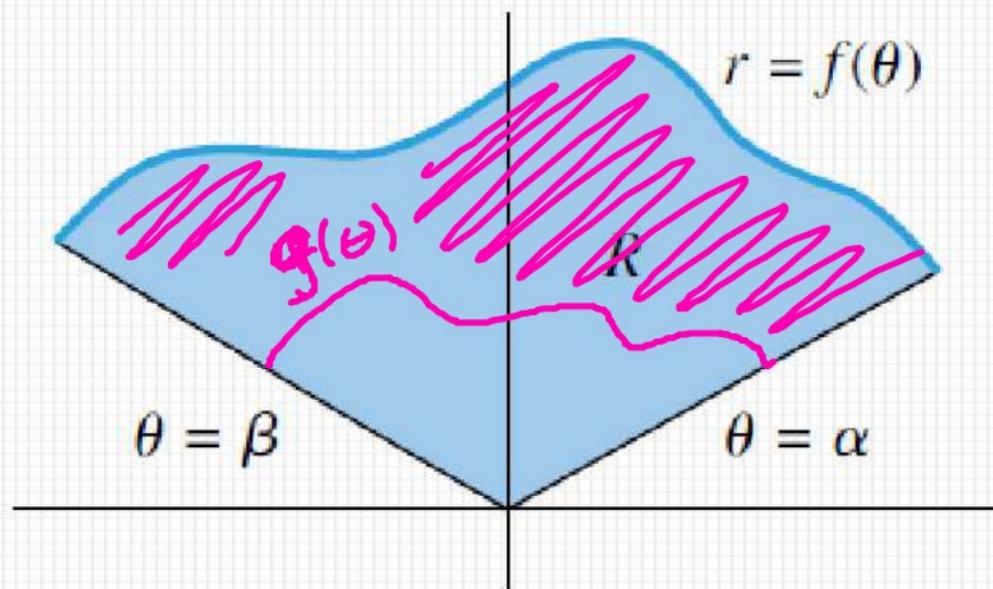


$$r = 2 \sin(2.15\theta)$$

$$0 \leq \theta \leq 16\pi$$

Greg Kelly, Hanford High School, Richland, Washington

Goal: Find an area contained by polar functions.



BIG AREA - SMALL

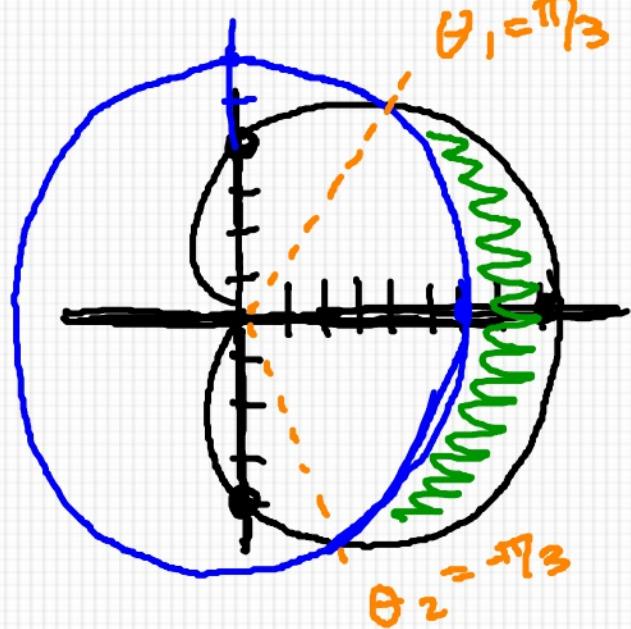
$$\frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 d\theta - \frac{1}{2} \int_{\alpha}^{\beta} [g(\theta)]^2 d\theta$$

$$\frac{1}{2} \int_{\alpha}^{\beta} [f(\theta)]^2 - [g(\theta)]^2 d\theta$$

$$R^2 - r^2$$



Ex: Find the area inside of $r = 4 + 4\cos\theta$ and outside of $r = 6$:



$$4 + 4\cos\theta = 6$$

$$\cos\theta = \frac{1}{2}$$

$$\theta_1 = \pi/3$$

$$2 \cdot \frac{1}{2} \int_{\theta_1}^{\theta_2} (4 + 4\cos\theta)^2 - 6^2 d\theta$$

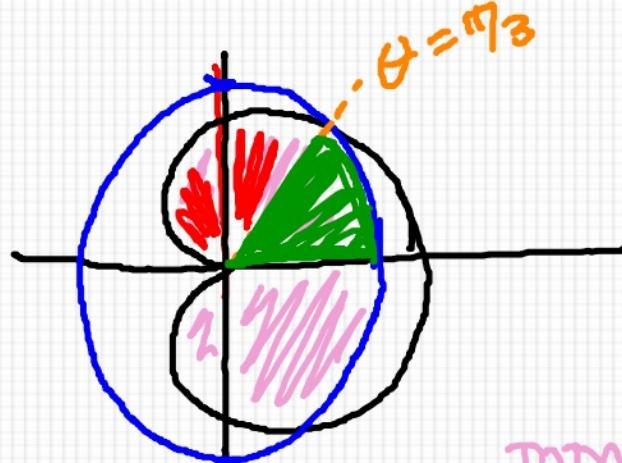
$$\approx 18.6105$$

OR

$$\int_0^{\pi/3} (r_1)^2 - (r_2)^2 d\theta$$



Ex: Find the area shared by $r_1 = 4 + 4\cos\theta$ and $r_2 = 6$:



$$\text{GREEN (sector)} : \frac{1}{6} \pi (6)^2 = 6\pi$$

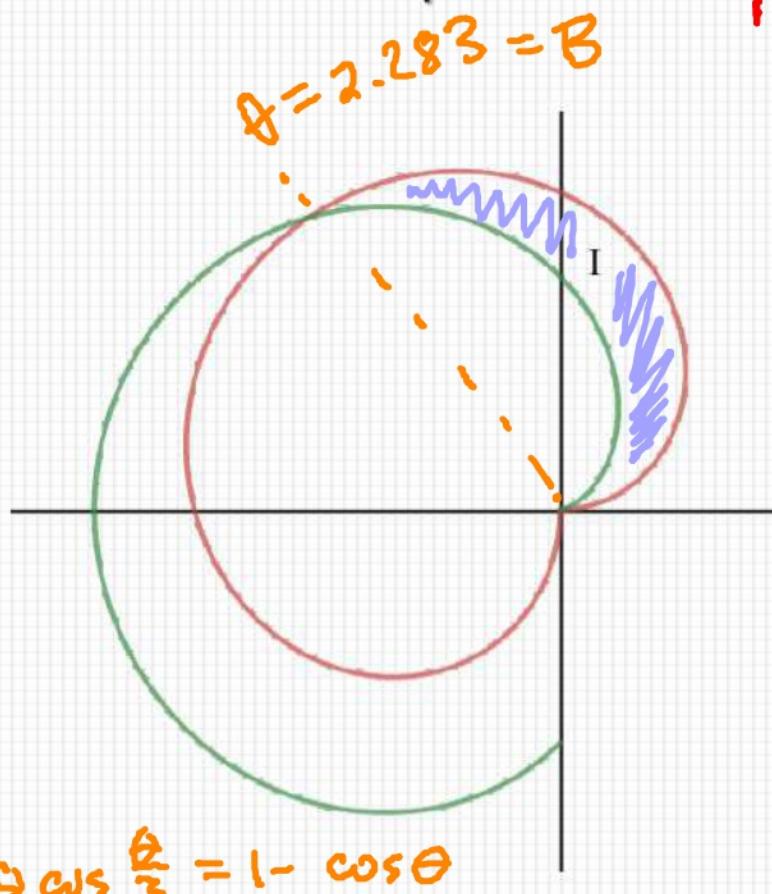
$$\text{RED (CARDIOID)} : \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} (4+4\cos\theta)^2 d\theta$$

$$\text{TOTAL} = \boxed{2(6\pi) + 2 \cdot \frac{1}{2} \int_{\frac{\pi}{3}}^{\pi} (4+4\cos\theta)^2 d\theta}$$

$$\approx 56.7877 \dots$$



Ex: Consider the polar curves $r_1 = \theta \cos\left(\frac{\theta}{3}\right)$ and $r_2 = 1 - \cos\theta$.

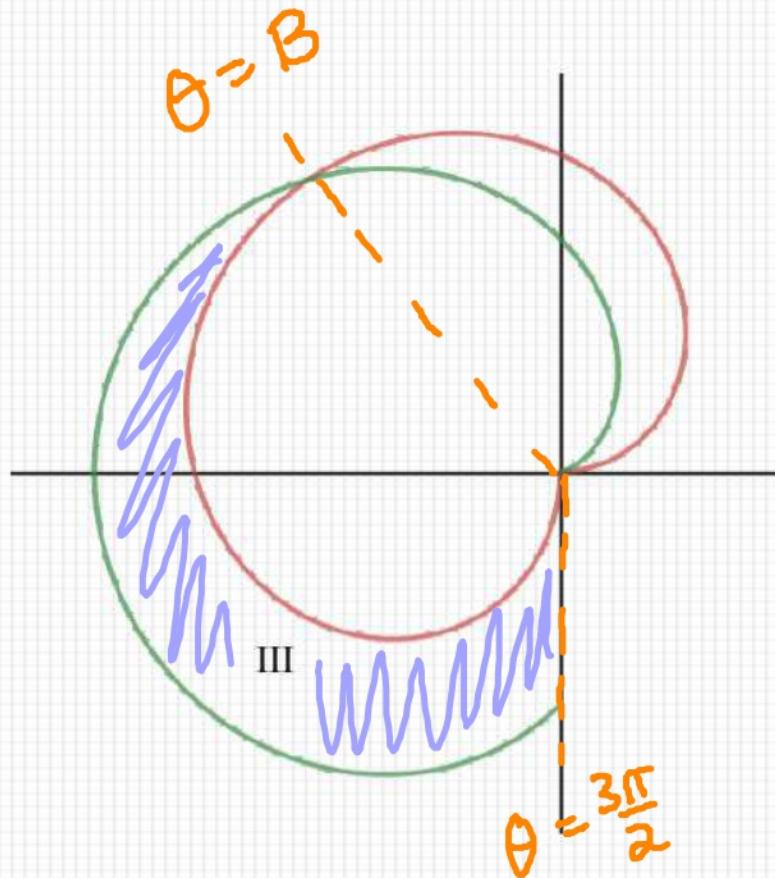


Find the area of region I.

$$\frac{1}{2} \int_0^B (\theta \cos \frac{\theta}{3})^2 - (1 - \cos \theta)^2 d\theta$$



Ex: Consider the polar curves $r_1 = \theta \cos\left(\frac{\theta}{3}\right)$ and $r_2 = 1 - \cos\theta$.

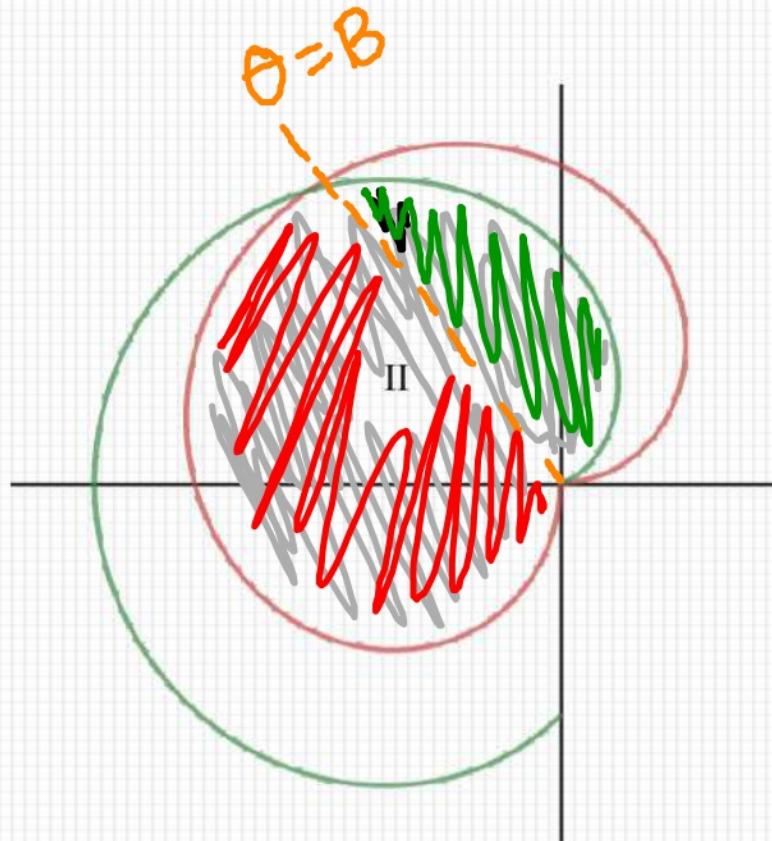


Find the area of region III.

$$\frac{1}{2} \int_{B}^{\frac{3\pi}{2}} (r_2)^2 - (r_1)^2 d\theta$$



Ex: Consider the polar curves $r_1 = \theta \cos\left(\frac{\theta}{3}\right)$ and $r_2 = 1 - \cos\theta$.

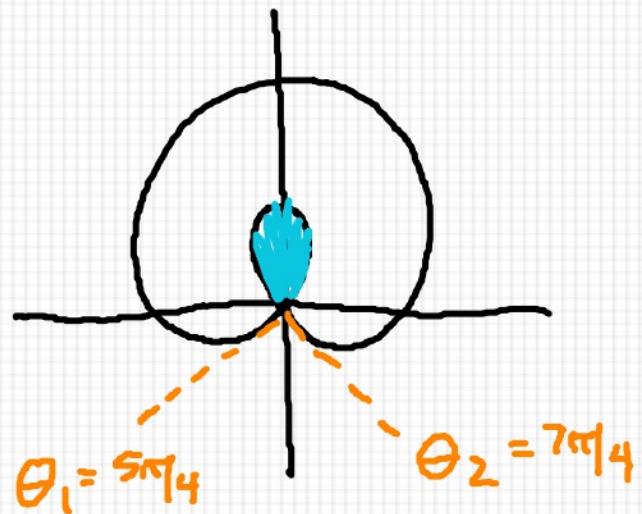


Find the area of region II.

$$\frac{1}{2} \int_0^B (r_2)^2 d\theta + \frac{1}{2} \int_B^{\frac{3\pi}{2}} (r_1)^2 d\theta$$



Ex: Consider the graph of $r = \sqrt{2} + 2\sin\theta$. Find the area within the inner loop.

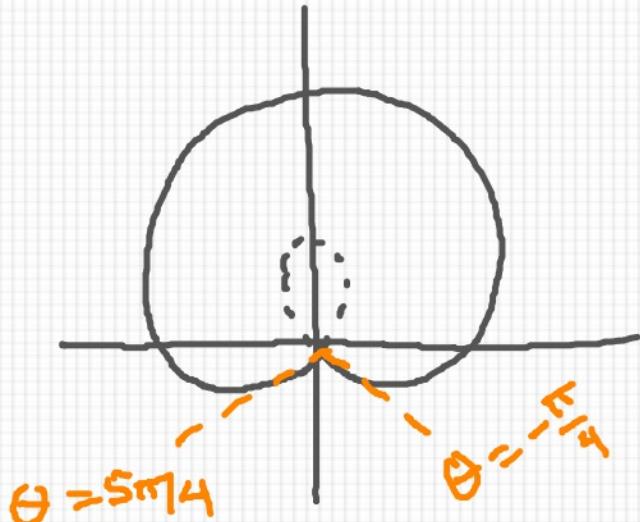


$$\frac{1}{2} \int_{5\pi/4}^{7\pi/4} (\sqrt{2} + 2\sin\theta)^2 d\theta$$

$$\sqrt{2} + 2\sin\theta = 0$$
$$\sin\theta = -\frac{\sqrt{2}}{2}$$
$$\theta_1 = \frac{5\pi}{4}, \frac{7\pi}{4}$$



Ex: Consider the graph of $r = \sqrt{2} + 2\sin\theta$. Find the area within the outer loop.



$$\frac{1}{2} \int_{-\pi/4}^{5\pi/4} (\sqrt{2+2\sin\theta})^2 d\theta$$

OR

$$2 \cdot \frac{1}{2} \int_{-\pi/4}^{\pi/2} (\sqrt{2+2\sin\theta})^2 d\theta$$

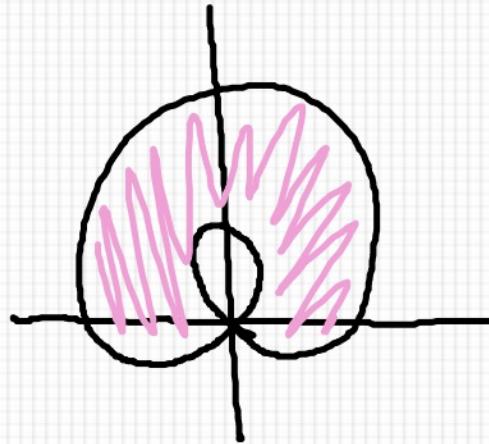
OR

$$\frac{1}{2} \int_0^{2\pi} (\sqrt{2+2\sin\theta})^2 d\theta$$

- PREVIOUS
PROBLEM



Ex: Consider the graph of $r = \sqrt{2} + 2\sin\theta$. Find the area between the inner loop and the outer loop.



SUBTRACT PREVIOUS TWO ANSWERS.

$$\frac{1}{2} \int_0^{2\pi} (\sqrt{2+2\sin\theta})^2 d\theta - 2 \cdot \frac{1}{2} \int_{\frac{5\pi}{4}}^{\frac{7\pi}{4}} (\sqrt{2+2\sin\theta})^2 d\theta$$

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Classwork:

Section 10.6 WS (Multiple Curves)

HW:

Anton p. 640 #4,5,6,7,10,12,13,14,15,18

EVENS : S.U.B.D.N.I.

